DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination – Winter 2018

Course: S.Y.B. Tech (All Branches) **Semester:** III

Subject Name: Engineering Mathematics-III **Subject Code:** BTBSC301

Max Marks:60 **Date:**30/11/2018 **Duration:** 03 Hrs

Instructions to the Students:

1. *Attempt **Any Five** questions of the following .All questions carry equal marks.*
2. *Use of non-programmable scientific calculators is allowed.*
3. *Figures to the right indicate full **Marks**.*

Q. 1. a) Show that,

$$\int_0^\infty \frac{\sin at}{t}\, dt = \frac{\pi}{2}.$$ **[4]**

b) Find the Laplace transform of

$$\int_0^t \frac{e^{-3u}\sin 2u}{u}\, du.$$ **[4]**

c) Find the Laplace transform of the function

$$f(t) = \begin{cases} 2 & , 0 < t < \pi \\ 0 & , \pi < t < 2\pi \\ \sin t & , \quad t > 2\pi \end{cases}$$ **[4]**

Q.2 . a) Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$. **[4]**

b) By convolution theorem, find inverse Laplace transform of

$$\frac{s}{(s^2+1)(s^2+4)}.$$ **[4]**

c) By Laplace transform method, solve the following simultaneous equations

$$\frac{dx}{dt} - y = e^t \; ; \; \frac{dy}{dt} + x = \sin t \; ; \; given\ that\ x(0) = 1, y(0) = 0.$$ **[4]**

Q. 3. a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0 & , |x| > 1. \end{cases}$$ **[4]**

b) Find the Fourier sine transform of $e^{-|x|}$, and hence show that

$$\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, \quad m > 0.$$ **[4]**

c) Using Parseval's Identity, prove that

$$\int_0^{\infty} \frac{t^2}{(t^2+1)^2}\, dt = \frac{\pi}{4}.$$ [4]

Q.4. a) Solve the partial differential equation

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$$ [4]

b) Use method of separation of variables to solve the equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u\ ;\ given\ that\ u(x,0) = 6e^{-3x}.$$ [4]

c) Find the temperature in bar of length $2\ units$ whose ends are kept at zero temperature and lateral surface insulated if initial temperature is

$$\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right).$$ [4]

Q. 5. a) If $f(z)$ is analytic function with constant modulus, show that $f(z)$ is constant. [4]

b) If the stream function of an electrostatic field is $\psi = 3xy^2 - x^3$, find the potential function ϕ, where $f(z) = \phi + i\psi$. [4]

c) Prove that the inversion transformation maps a circle in the z-plane into a circle in w-plane or to a straight line if the circle in the z-plane passes through the origin. [4]

Q.6. a) Evaluate $\oint_c \frac{e^z}{(z-2)}\, dz$, where c is the circle $|z| = 3$. [4]

b) Evaluate $\oint_c \tan z\, dz$, where c is the circle $|z| = 2$. [4]

c) Evaluate, using Cauchy's integral formula : [4]

1) $\oint_c \frac{\cos(\pi z)}{(z^2-1)}\, dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$.

2) $\oint_c \frac{\sin^2 z}{(z-\frac{\pi}{6})^3}\, dz$, where C is the circle $|z| = 1$.

*** **End** ***