

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination – May 2019

Course: B. Tech

Sem: III

Subject Name: Engineering Mathematics-III

Subject Code: BTBSC301

Max Marks: 60

Date: 28-05-2019

Duration: 3 Hr.

Instructions to the Students:

1. Solve ANY FIVE questions out of the following.
2. The level question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in () in front of the question.
3. Use of non-programmable scientific calculators is allowed.
4. Assume suitable data wherever necessary and mention it clearly.

	(Level/CO)	Marks
Q. 1 Attempt any three.		12
A) Find $L\{f(t)\}$, where $f(t) = t^2 e^{-3t} \sin t$	Understand	4
B) Express $f(t)$ in terms of Heaviside's unit step function and hence find its Laplace transform where $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$	Understand	4
C) Find $L\{f(t)\}$, where $f(t) = 2^t \int_0^t \frac{\sin 3u}{u} du$	Understand	4
D) By using Laplace transform evaluate $\int_0^\infty e^{-t} \left(\frac{1 - \cos 2t}{t} \right) dt$	Evaluation	4
Q. 2 Attempt the following.		12
A) Using convolution theorem find $L^{-1} \left\{ \frac{s^2}{(s^2+4)^2} \right\}$	Application	4
B) Find $L^{-1}\{\bar{f}(s)\}$, where $\bar{f}(s) = \cot^{-1} \left(\frac{s+3}{2} \right)$	Application	4
C) Using Laplace transform solve $y'' - 3y' + 2y = 12e^{-2t}$; $y(0) = 2$, $y'(0) = 6$	Application	4
Q. 3 Attempt any three.		12
A) Express $f(x) = \begin{cases} 1, & 0 \leq x \leq \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral and hence deduce that $\int_0^\infty \frac{1 - \cos \pi \lambda}{\lambda} \sin \pi x d\lambda = \frac{\pi}{4}$.	Evaluation	4
B) Using Parseval's identity for cosine transform, prove that	Application	4
$\int_0^\infty \frac{\sin at}{t(a^2+t^2)} dt = \frac{\pi}{2} \left(\frac{1 - e^{-a^2}}{a^2} \right)$		

- C) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \text{if } |x| \leq 1 \\ 0, & \text{if } |x| > 1 \end{cases}$. Hence prove **Understand** 4
that $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx = -\frac{3\pi}{16}$

- D) Find Fourier sine transform of $5e^{-2x} + 2e^{-5x}$ **Understand** 4

Q. 4 Attempt the following. 12

- A) Form the partial differential equation by eliminating arbitrary function f from $f(x + y + z, x^2 + y^2 + z^2) = 0$ **Synthesis** 4

- B) Solve $xz(z^2 + xy)p - yz(z^2 + xy)q = x^4$ **Analysis** 4

- C) Find the temperature in a bar of length two units whose ends are kept at zero temperature and lateral surface insulated if the initial temperature is **Application** 4
 $\sin \frac{\pi x}{2} + 3 \sin \frac{5\pi x}{2}$.

Q. 5 Attempt Any three. 12

- A) If the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic, find the values of the constants a, b, c and d . **Understand** 4

- B) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant. **Understand** 4

- C) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$. **Understand** 4

- D) Prove that the function $u = e^x(x \cos y - y \sin y)$ satisfies the Laplace's equation. Also find the corresponding analytic function. **Synthesis** 4

Q. 6 Attempt ANY TWO of the following. 12

- A) Evaluate $\oint_C \frac{z+4}{z^2+2z+5} dz$, where C is the circle $|z + 1 - i| = 2$. **Evaluation** 6

- B) Find the residues of $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$. **Understand** 6

- C) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. **Evaluation** 6

*** End ***