

Winter Examination – 2022

Course: - B. Tech.

Branch: - Common for All branches

Semester:- III

Subject Code & Name: BTBS301

Engineering Mathematics-III

Max. Marks: - 60

Date: - 09/03/2023

Duration: - 3-Hrs

Instructions to the Students:

1. All the questions are compulsory.
2. The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in () in front of the question.
3. Use of non-programmable scientific calculators is allowed.
4. Assume suitable data wherever necessary and mention it clearly.

	(Level/CO)	Marks
Q. 1 Solve Any Three of the following.		12
A) Find Laplace Transform of $e^{-3t} \sin^2 t$	L3/CO1	4
B) Find Laplace Transform of $f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$ where $f(t)$ is periodic function of period 2.	L3/CO1	4
C) Evaluate using Laplace Transform.: $\int_0^\infty \frac{\cos 4t - \cos 3t}{t} dt$	L3/CO1	4
D) Find Laplace Transform of $(1 + 2t - 3t^2 + 4t^3)H(t - 2)$	L3/CO1	4
Q2 Solve Any Three of the following.		12
A) Find the inverse Laplace transformation of the function. $\log \left(1 + \frac{a^2}{s^2} \right)$	L3/CO2	4
B) By using convolution theorem find $L^{-1} \left[\frac{s}{(s^2+4)(s^2+9)} \right]$	L3/CO2	4
C) Find the inverse Laplace transformation of the function. $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$	L3/CO2	4
D) Solve using Laplace transformation $y'' + 3y' + 2y = t\delta(t - 1)$ for which $y(0) = y'(0) = 0$	L3/CO2	4

Q.3 Solve Any Three of the following.**(12)**

A) Using Parseval's identity prove that $\int_0^\infty \frac{x^2}{(x^2+1)^2} dx = \frac{\pi}{4}$ L3/CO3 **4**

B) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0 & , \quad |x| > 1 \end{cases} \quad \text{L3/CO3} \quad \mathbf{4}$$

C) Find the Fourier Sine transform e^{-ax} , $a > 0$ L3/CO3 **4**

D) Find the Fourier cosine transform of the function $f(y) = \begin{cases} \cos y, & 0 < y < a \\ 0, & y > a \end{cases}$ L3/CO3 **4**

Q.4 Solve Any Three of the following.**(12)**

A) Form the partial differential equation by eliminating arbitrary constants from L3/CO4 **4**

$$(x - a)^2 + (y - b)^2 = z^2 \cot^2 \alpha$$

B) Solve the Partial differential equation $x(y - z)p + y(z - x)q = z(x - y)$ L3/CO4 **4**

C) Use the method of separation of variables to solve

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u \quad \text{given that } u(x, 0) = 6e^{-3x} \quad \text{L3/CO4} \quad \mathbf{4}$$

D) A bar with insulated at its ends is initially at temperature 0°C throughout. The end $x = 0$ is kept at 0°C for all times and the heat is suddenly applied so that $\frac{\partial u}{\partial x} = 10$ at $x = t$ for all time. Find the temperature function $u(x, t)$ L3/CO4 **4**

Q.5 Solve Any Three of the following.**(12)**

A) Determine k such that the function $f(z) = e^x \cos y + ie^x \sin ky$ is analytic. L3/CO5 **4**

B) Show that $u = x^2 - y^2 - 2xy - 2x + 3y$ is a harmonic function and L3/CO5 **4**
hence determine the analytic function $f(z)$ in terms of z .

C) Determine the pole of the function $f(z) = \frac{2z-1}{z(z+1)(z-3)}$ and also find the residue at each pole

& sum of all residues. L3/CO5 **4**

D) Evaluate L3/CO5 **4**

$$\oint_C \frac{\sin \pi z^2 + 2z}{(z-1)^2(z-2)} dz, \text{ Where } C \text{ is the circle } |z| = 4$$

*** End ***