	DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE **Regular & Supplementary Winter Examination-2023** **Course: B. Tech.** **Branch: ALL** **Semester: III** **Subject Code & Name: BTBS301/ BTES 301 Engineering Mathematics-III** **Max Marks: 60** **Date: 02.01.2024** **Duration: 3 Hr.**		
	Instructions to the Students: 1. *All the questions are compulsory.* 2. *The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.* 3. *Use of non-programmable scientific calculators is allowed.* 4. *Assume suitable data wherever necessary and mention it clearly.*		
		(Level/CO)	Marks
Q. 1	**Solve Any Two of the following.**		**12**
A)	Find the Laplace transform of $f(t) = t^2 \sin 2t$	**Understand/ CO1**	**6**
B)	Find Laplace transform of $F(t) = \int_0^t \frac{e^{-at}-e^{-bt}}{t} dt$	**Understand /CO1**	**6**
C)	Find the Laplace transforms of $f(t) = \frac{t}{T}$, for $0 < t < T$ (saw - tooth wave function of period T)	**Apply/CO1**	**6**
Q.2	**Solve Any Two of the following.**		**12**
A)	Find inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$	**Understand /CO2**	**6**
B)	By using Partial fraction expansion to find inverse Laplace transform of $F(s) = \frac{s}{(s^2+1)(s^2+4)}$	**Understand /CO2**	**6**
C)	Using the Laplace transform, solve the differential equation $\frac{d^2x}{dt^2} + 9x = \cos 2t$; if $x(0) = 1,\ x\left(\frac{\pi}{2}\right) = -1.$	**Apply/CO2**	**6**
Q. 3	**Solve Any Two of the following.**		**12**
A)	Express the function $f(x) = \begin{cases} 1 \text{ for } \lvert x\rvert \le 1 \\ 0 \text{ for } \lvert x\rvert > 1 \end{cases}$ as a Fourier integral.	**Understand /CO3**	**6**
B)	Find the Fourier sine transform of $f(x) = e^{-\lvert x\rvert}$, and hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2},\ m > 0.$	**Understand /CO3**	**6**
C)	Using Parseval's identity, show that $\int_0^\infty \frac{t^2}{(4+t^2)(9+t^2)} dt = \frac{\pi}{10}$	**Apply/CO3**	**6**
Q.4	**Solve Any Two of the following.**		**12**
A)	Solve the following partial differential equations $(mz - ny)p + (nx - lz)q = ly - mx$	**Understand /CO4**	**6**

B)	A string is stretched and fastened to two points $\boldsymbol{l}$ apart. Motion is started by replacing the string in the form $\boldsymbol{y = A\sin\frac{\pi x}{l}}$ from which it is released at time $\boldsymbol{t = 0}$. Show that the displacement of a point at a distance x from one end at time t is given by $\boldsymbol{y(x,t) = A\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}}$.	**Apply/CO4**	**6**
C)	Solve the following equation by the method of separation of variables: $\boldsymbol{\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x}$, given that $\boldsymbol{u = 0}$ when $\boldsymbol{t = 0}$ and $\boldsymbol{\frac{\partial u}{\partial t} = 0}$ when $\boldsymbol{x = 0}$.	**Apply /CO4**	**6**
Q. 5	**Solve Any Two of the following.**		**12**
A)	If $\boldsymbol{f(z)}$ is analytic, show that $\left[\frac{\partial\|f(z)\|}{\partial x}\right]^2 + \left[\frac{\partial\|f(z)\|}{\partial y}\right]^2 = \|f'(z)\|^2$.	**Understand /CO5**	**6**
B)	Apply Cauchy's integral Formula to evaluate $\oint_C \frac{e^{-z}}{z+1}\,dz$, where **C** is the circle **(a)** $\|z\| = 2$ and **(b)** $\|z\| = \frac{1}{2}$.	**Apply/CO5**	**6**
C)	State Cauchy's residue theorem and evaluate $\oint_C \tan z\, dz$, where C is the circle $\|z\| = 2$.	**Apply /CO5**	**6**
	***** End *****		